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# Estimating Promotion Probabilities of Navy Officers Based on Individual's Attributes and Other Global Effects

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> Reviewed by David M. Cashbaugh Director

Approved and released by David M. Cashbaugh Director

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#### **Foreword**

This effort was funded by the Office of Naval Research (ONR) and Deputy Chief of Naval Operations (N-1). The basic objective of the project was to develop a promotion probability model for the individual officers as well as to provide a forecasting tool for the policy maker. The authors wish to thank the funding sponsor, Office of Naval Research and the Officer Community Management (OCM) Division, BUPERS -31, for their assistance in this project.

DAVID M. CASHBAUGH Director

#### **Summary**

The basic objective of the CCMT Officers project is to develop an econometric model that captures the promotion trajectory for the individual officers, conditional on past behavior and performance as well as on past (and current) economic and policy conditions. The model is also conditional on staying in the Navy the minimal time for promotion. The estimated results are also used as a forecasting tool for the policy maker. The econometric framework used here is a two-step first order Markov model that accommodates time dependent information, cohort information, censoring problems with the data as well as incorporating macro economic and policy level information. In the first step, the conditional probabilities of staying or leaving the Navy are estimated conditional on staying in the Navy enough time to be eligible for promotion. In the second step of the estimation, the promotion probability within the next year (conditional on all the independent variables) is estimated. The estimation model is an Information-Theoretic, Generalized Maximum Entropy model that is nonparametric in the distribution.

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#### **Introduction and Objectives**

The basic objective of the Officers promotion modeling project was to develop a promotion probability model for individual officers as well as to provide a forecasting tool for the policy maker. In a previous project, a theoretical transition probability model was developed. It is a first order Markov model that accommodates time dependent information, cohort information, censoring problems with the data as well as incorporating macro economic and policy level information. The estimation model specified for this project was an Information-Theoretic, Generalized Maximum Entropy model that is a semi-parametric model. The Markov model developed here is a two-step model. In the first step, the probability of staying or leaving the Navy for a period of time (based on Navy rules specified on the years since commission) is calculated. Conditional on that, in the second step the promotion probability is estimated.

The promotion structure of officers is very different than that of the enlisted personnel. Therefore, an important aspect of this project was to test whether an econometric model can explain much of the promotions of officers. We find here, that it is possible. There are three sets of complete estimates for each skill group analyzed. The first is based on the complete data set. The second is based on all data prior to September 11, 2001 (peace period). The third is based on all data after September 11, 2001 (war period).

In the Section 2 the basic Markov model is defined and described. The estimator is described in Section 3. Some of the basic statistics are discussed as well in that Section. The data and basic analysis are described in Section 4. To keep this report a reasonable size, the basic results for the Aviation group are summarized here as an example. These results demonstrate the capabilities of the estimations done and the resulting forecasts. Appendix A provides a summary of the data used in the final estimates and some of the models investigated.

#### The Basic Markov Model

#### **The Basic Promotion Model**

The best model to capture the officers' promotion is the two-step first order Markov transition probabilities model. Conditional on a first step that estimates the conditional probability of staying in the Navy, the transition/promotion process is as follows.

Let  $y_{iij}$  be state j (grade level j=03, ..., 06) of individual i in period t. Specifically, for each individual i (i=1, 2, ..., n) let  $y_{iij} = 1$  if state j (j=1, 2, ..., K) is observed at period t (t=1, 2, ..., T), and  $y_{iij} = 0$  for all other K-1 states. Next, define the  $K \times K$  matrix of

<sup>&</sup>lt;sup>1</sup> Unpublished technical report; 'Career Case Manager Technologies for Enlisted Personnel'

transition (promotion) probabilities  $P = (p_{kj})$  representing the probability of promotion from state (grade) k to state (grade) k+1=j. One of the K states represents leaving the navy (it can be refined to voluntary and non-voluntary exits). Since an individual can be promoted just one grade at a time, and assuming no individual is being demoted to a lower grade (note that in the empirical model we allow for demotion), in this model  $p_{kj} = 0$  for all j < k, and for all j > k+1.

The basic relationship between period (t-1) and period t is captured via the  $K \times K$  matrix of transition (promotion) probabilities

$$y_{iij} = \sum_{\{y_{i,t-1,k}=0\}} p_{kj} y_{i,t-1,k} + \sum_{\{y_{i,t-1,k}\neq 0\}} p_{kj} y_{i,t-1,k} \equiv \sum_{k=1}^{K} p_{kj} y_{i,t-1,k}$$
(1)

where the middle two terms represent the two subsets of the data: the case where previous (K-1) states are zero and the observed k state.

Taking into account the noise in the observed data, the correct noisy observed model is

$$y_{iij} = \sum_{\{y_{i,t-1,k}=0\}} p_{kj} y_{i,t-1,k} + \sum_{\{y_{i,t-1,k}\neq0\}} p_{kj} y_{i,t-1,k} + \varepsilon_{iij} \equiv \sum_{k=1}^{K} p_{kj} y_{i,t-1,k} + \varepsilon_{iij}$$
(2)

where  $\varepsilon_{iij}$  represents the noise in the data. In this model, each  $\varepsilon_{iij}$  is naturally bounded in [-1, 1], with expected mean value of zero. We allow correlations across time in that model. The exact covariance structure and the noise structure will be finalized and discussed in the empirical part.

#### **Introducing the Individual Level Information (Covariates)**

There are two types of individual level covariates in this model: Time Dependent and Time Independent.

- 1. Let the time-dependent variables (covariates) be  $x_{iil}^0$  for l=1,2,...,L. These variables capture the information that may change over time, such as the individual's evaluations, the individual's standing within her/his cohort, previous jobs, previous behavioral problems, changes in personal characteristics (married, children, additional education, etc.).
- 2. Let the time-independent variables (covariates) be  $x_{ih}^1$  for h=1,2,...,H. These variables capture the information that does not change over time, including the individual's basic characteristics (AFQT, race, gender, basic education<sup>2</sup> level, etc.).

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<sup>&</sup>lt;sup>2</sup> Note that education may change over time, and in that case it is added as a time-dependent variable.

Next, we define  $X = [X^0 X^1]$  as the matrix of <u>all</u> of the individual-level variables (demographics, socio-economic and past "behavior" and performance in the Navy). This matrix is of dimension  $(N \times T) \times (L + H) = (N \times T) \times S$  for S = L + H and S = 1, 2, ..., L, L + 1, ..., S. There is a total of S individual level variables, where the last H variables do not vary over time for each individual I = 1, 2, ..., N.

To capture the relationship between the observed data, y, the unknown probabilities P, and the covariates X, we introduce the following (cross moments) relationship:

$$\sum_{t=2}^{T} \sum_{i=1}^{N} y_{itj} x_{its} = \sum_{t=2}^{T} \sum_{i=1}^{N} \sum_{\{y_{i,t-1,k}=0\}}^{K} p_{kj} x_{i,t-1,s} + \sum_{t=2}^{T} \sum_{i=1}^{N} \sum_{\{y_{i,t-1,k}\neq0\}}^{K} p_{kj} y_{i,t-1,k} x_{i,t-1,s} + \sum_{t=2}^{T} \sum_{i=1}^{N} \varepsilon_{i,t-1,j} x_{i,t-1,s}$$

$$\equiv \sum_{t=1}^{T-1} \sum_{i=1}^{N} \sum_{\{y_{iik}=0\}}^{K} p_{kj} x_{its} + \sum_{t=1}^{T-1} \sum_{i=1}^{N} \sum_{\{y_{iik}\neq0\}}^{K} p_{kj} y_{itk} x_{its} + \sum_{t=1}^{T-1} \sum_{i=1}^{N} \varepsilon_{itj} x_{its}$$

$$= \sum_{t=1}^{T-1} \sum_{i=1}^{N} \sum_{k=1}^{K} p_{kj} y_{itk} x_{its} + \sum_{t=1}^{T-1} \sum_{i=1}^{N} \varepsilon_{itj} x_{its}$$
(3)

A discussion of all variables is provided below.

#### Introducing the Global/Macro-Level Variables

The next step is to extend this framework to include economic and political variables. These variables may have direct or indirect effects on the individuals' promotion probabilities thorough their global effect on the navy, the individual's specific skill (or profession), or the environment as a whole.

Specifically, let  $\mathbf{z}_g$  be a G-dimensional vector of global variables for each period t, composed of G macro and policy variables (or instruments). As we do not know the direct impact and relationship between these set of variables and our data  $\mathbf{y}$ , the covariates X, or the promotion probabilities P, we introduce this information via the cross-moments of the global and the individual-level variables. There are two ways to do so. The first one is the more general case which allows interaction between these global quantities and each one of the individual's characteristics. The second case is done by incorporating these global variables into the X matrix. After testing the data, in the empirical estimation we use the second case which is described below.

Define  $X = [X^0 X^1 Z]$  as the matrix of all of the variables (individual level variables, the X's, and global variables, the Z's). This matrix is of dimension

$$(N \times T) \times (L + H + G) = (N \times T) \times S^{g}$$
 for  $S^{g} = L + H + G$  and  $s = 1, 2, ..., L, L + 1, ..., L + H, ...$ 

L+H+1,..., L+H+G. There are a total of  $S^s$  variables. Rewriting Eq. (3) yields

$$\sum_{t=2}^{T} \sum_{i=1}^{N} y_{itj} x_{its^g} = \sum_{t=1}^{T-1} \sum_{i=1}^{N} \sum_{k=1}^{K} p_{kj} y_{itk} x_{its^g} + \sum_{t=1}^{T-1} \sum_{i=1}^{N} \varepsilon_{itj} x_{its^g} .$$
 (3A)

The global-macro level variables enter as lag variables (with one to three periods lag, and as differenced lag variables) where, again the empirical results will determine the exact variables used.

#### **The Estimation Method**

#### The Information-Theoretic, Generalized Maximum Entropy Basic Model

To simplify the presentations and notations, we formulate the basic model with covariates (or the second case presented in 3A above). Following the Information Theoretic (IT) - Generalized Maximum Entropy (GME) approach we first, reformulate the noise components  $\varepsilon_{iij}$  to be proper probabilities defined on some support space  $\mathbf{v}$ . For background on the GME and related work see Golan, Judge and Miller (1996) and Golan, Judge and Perloff (1996). For background and recent work on IT and its relationship to GME and other methods of estimation and inference see special issues of the Journal of Econometrics (2002, 2007). Rewriting Eq. (3A) yields

$$\sum_{t=2}^{T} \sum_{i=1}^{N} y_{itj} x_{its} = \sum_{t=1}^{T-1} \sum_{i=1}^{N} \sum_{k=1}^{K} p_{kj} y_{itk} x_{its} + \sum_{t=1}^{T-1} \sum_{i=1}^{N} x_{its} \mathcal{E}_{itj}$$

$$= \sum_{t=1}^{T-1} \sum_{i=1}^{N} \sum_{k=1}^{K} p_{kj} y_{itk} x_{its} + \sum_{t=1}^{T-1} \sum_{i=1}^{N} \sum_{m=1}^{M} x_{its} w_{itjm} v_{m}$$

$$(4)$$

with  $\sum_{m=1}^{M} w_{itjm} = 1$  and where  $\varepsilon_{itj} \equiv \sum_{m=1}^{M} w_{itjm} v_m$  for  $M \ge 2$ . Since  $\varepsilon_{itj} \in [-1,1]$  for all i, t, j, then  $v_m \in [-1,1]$  and  $\boldsymbol{v}$  is a symmetric around zero support space for each random error defined above. In the empirical analysis, examples and discussions of the size of M as well as using continuous support spaces will be provided.

By now, we have reformulated the basic Markov model to include all of the available information (personal and global). We also have converted the unknown errors to be fully represented by a set of proper probabilities (W), so all of the unknown quantities here (P and W) are proper probability distributions. We can now construct the GME estimation method which maximizes the joint Shannon (1948) entropies of the signal, P, and the noise, W, subject to the available information (the data) and the requirement that both P and W are proper probability distributions.

The basic GME Markov model is

$$Max \left\{ -\sum_{k,j} p_{kj} \log p_{kj} - \sum_{itjm} w_{itjm} \log w_{itjm} \right\}$$
$$\{p, w\}$$

$$\sum_{t=2}^{T} \sum_{i} y_{itj} x_{its} = \sum_{t=1}^{T-1} \sum_{i} \sum_{k=1}^{K} p_{kj} y_{itk} x_{its} + \sum_{t=1}^{T-1} \sum_{i} \sum_{m} w_{itjm} x_{its} v_{m}$$

$$\sum_{j} p_{kj} = 1; \sum_{m} w_{itjm} = 1$$

The GME solution is

$$\hat{p}_{kj} = \frac{\exp\left(-\sum_{t=1}^{T-1} \sum_{i,s} y_{itk} x_{its} \hat{\lambda}_{js}\right)}{\sum_{j} \exp\left(-\sum_{t=1}^{T-1} \sum_{i,s} y_{itk} x_{its} \hat{\lambda}_{js}\right)} = \frac{\exp\left(-\sum_{t=1}^{T-1} \sum_{i,s} y_{itk} x_{its} \hat{\lambda}_{js}\right)}{\Omega_{k}}$$
(6a)

and

$$\hat{w}_{itjm} = \frac{\exp\left(-\sum_{s} x_{its} v_{m} \hat{\lambda}_{js}\right)}{\sum_{m} \exp\left(-\sum_{s} x_{its} v_{m} \hat{\lambda}_{js}\right)} \equiv \frac{\exp\left(-\sum_{s} x_{its} v_{m} \hat{\lambda}_{js}\right)}{\Psi_{itj}}$$
(6b)

where  $\hat{\lambda}_{js}$  are the  $J \times S$  estimated Lagrange multipliers associate with the data (Eq. 4), and the estimated noise components are  $\hat{\varepsilon}_{iij} = \sum_m \hat{w}_{iijm} v_m$ .

#### The concentrated (dual) GME method is

Instead of using the constrained optimization estimation model (5), the GME can be formulated as an unconstrained, concentrated (or a generalized likelihood) model:

$$\ell(\lambda) = \sum_{t=2}^{T} \sum_{j=1}^{K} \sum_{i,s} y_{itj} x_{its} \lambda_{sj} + \sum_{k} \log \left[ \sum_{j} \exp \left( -\sum_{t=1}^{K} \sum_{i,s} y_{itk} x_{its} \lambda_{sj} \right) \right] + \sum_{i,t,j} \log \left[ \sum_{m} \exp \left( -\sum_{s} x_{its} v_{m} \lambda_{sj} \right) \right]$$

$$= \sum_{t=2}^{T} \sum_{j=1}^{K} \sum_{i,s} y_{itj} x_{its} \lambda_{sj} + \sum_{k} \log \Omega_{k} \left( \right) \log \sum_{i,t,j} \Psi_{ij} \left( \right)$$

$$(7)$$

where  $\lambda_{js}$  are the  $J \times S$  Lagrange multipliers which are the real set of unknown and unobserved quantities in this model. Minimizing (7) and solving for  $\lambda$ , yields the estimated  $\hat{\lambda}$ , which in turn yield the optimal probabilities  $\hat{p}_{kj}$  and  $\hat{w}_{itjm}$  via relationship (6)3.

#### **Introducing Prior Information**

Within the approach taken here, it is easy to introduce prior information representing some prior knowledge or belief of the Markov probabilities P, call it  $P^0$ . This is done by substituting the entropy objective with the cross-entropy or Kulback-Liebler information-divergence measure. Rewriting the objective of our GME estimator as

$$I(P,W;P^{0},W^{0}) = \sum_{k,j} p_{kj} \log(p_{kj}/p_{kj}^{0}) + \sum_{itjm} w_{itjm} \log(w_{itjm}/w_{itjm}^{0})$$
(9)

and minimizing I(P, W) with respect to (3A or 4) and the requirements of proper probabilities ( $\sum_{j} p_{kj} = 1$ ;  $\sum_{m} w_{iijm} = 1$ ) yields the optimal solutions for the Generalized Cross Entropy (GCE) model

$$\tilde{p}_{kj} = \frac{p_{kj}^{0} \exp\left(\sum_{t=1}^{T-1} \sum_{i,s,g} y_{iik} x_{its} z_{tg} \tilde{\lambda}_{jsg}\right)}{\sum_{j} p_{kj}^{0} \exp\left(\sum_{t=1}^{T-1} \sum_{i,s,g} y_{iik} x_{its} z_{tg} \tilde{\lambda}_{jsg}\right)} = \frac{p_{kj}^{0} \exp\left(\sum_{t=1}^{T-1} \sum_{i,s,g} y_{iik} x_{its} z_{tg} \tilde{\lambda}_{jsg}\right)}{\Omega_{k}}$$

$$(10A)$$

and

-

<sup>&</sup>lt;sup>3</sup> To keep this report from expanding too much, the relevant statistics as well as the variance-covariance matrix will be presented with the empirical findings. However, these quantities are available upon request.

$$\tilde{w}_{itjm} = \frac{w_{itjm}^{0} \exp\left(\sum_{s,g} x_{its} z_{tg} v_{m} \tilde{\lambda}_{jsg}\right)}{\sum_{m} w_{itjm}^{0} \exp\left(\sum_{s,g} x_{its} z_{tg} v_{m} \tilde{\lambda}_{jsg}\right)} \equiv \frac{w_{itjm}^{0} \exp\left(\sum_{s,g} x_{its} z_{tg} v_{m} \tilde{\lambda}_{jsg}\right)}{\Psi_{itj}}$$
(10A)

where now we introduced the macro level variables Z in the more general framework ("first case" discuss in Section 2.3). Finally,  $\tilde{\varepsilon}_{itj} = \sum_{m} \tilde{w}_{itjm} v_m$ . Note that the priors for the

noise terms are *always taken to be uniform* and are incorporated here just for generalization.

The dual, concentrated GCE is just

$$\ell(\lambda) = \sum_{t=2}^{T} \sum_{j=1}^{K} \sum_{i,s,g} y_{itj} x_{its} z_{tg} \lambda_{sjg} - \sum_{k} \log \left[ \sum_{j} p_{kj}^{0} \exp \left( \sum_{t=1}^{T} \sum_{i,s,g} y_{itk} x_{its} z_{tg} \lambda_{sjg} \right) \right]$$

$$- \sum_{i,t,j} \log \left[ \sum_{m} w_{itjm}^{0} \exp \left( \sum_{s,g} x_{its} z_{tg} v_{m} \lambda_{sjg} \right) \right]$$

$$= \sum_{t=2}^{T} \sum_{j=1}^{K} \sum_{i,s,g} y_{itj} x_{its} z_{tg} \lambda_{sjg} - \sum_{k} \log \Omega_{k} \left( \right) \log \left( \sum_{i,t,j} \Psi_{itj} \left( \right) \right)$$

$$(11)$$

where both normalization factors  $\Omega_k(\lambda)$  and  $\Psi_{ii}(\lambda)$  are defined in (10A-10B) above.

#### **Forecasting and Other Statistics**

Based on the estimated parameters, we can (1) forecast the mean behavior of the skill group analyzed many periods into the future; (2) forecast each officer's potential promotion trajectory, for many periods into the future, based on her/his personal characteristics, past performance and other macro/policy variables; (3) forecast the individuals (or group level) promotion trajectories under certain constraints, such as geographic restrictions; and (4) simulate different policy and other global scenarios, such as the impact of expanding/shrinking a certain skill group, or the impact of a booming economy on the promotion trajectories. Technically, these forecasts are a simple multiplication of the promotion matrix while accommodating for the time varying variables. For example, if P is the 5 by 5 matrix of promotion probability (with the states: O3, O4, O5, O6 and Loss), then  $P \times P$  is a 5 by 5 promotion probabilities two periods into the future.

The impact of each one of the right hand side variables on the promotion probabilities of each individual sailor are captured via the Marginal Effects. Technically, the Marginal Effects of individual *i*, at period *t*, are:

$$\frac{\partial p_{kj}}{\partial x_{iis}} = p_{kj} y_{iik} \left( \lambda_{sj} - \sum_{j} p_{kj} \lambda_{sj} \right).$$

#### **Empirical Results**

#### **Data and Analysis**

Five officers' skill groups are analyzed here: Aviation (AVI), Surface Warfare Group (SWO), Special Operations (SPOPS), Special Warfare (SPWAR), and Submarines (SUB). The dates for the data used for each skill group are:

Skill Group Start End Aviation Jan 96 Aug 06 **SWO** Jan 96 Sep 06 Jan 96 Mar 07 Submariner SP\_OPS Jan 96 Apr 07 SP\_WAR Jan 96 Apr 07

Table.1. Officer Skill Groups

For each skill group we provide estimates based on the full data set (with a dummy variable for September, 2001), and on the two mutually exclusive subsets of Pre and Post September 2001. This is because our basic hypothesis here is that the basic set of preferences and policies have changed after September, 2001. Therefore, it is more accurate to look at that data separately. Because the method of estimation used here allows us to analyze relatively smaller samples, we are able to do it.

The basic variables used in all the models are summarized in the following table (Table 4.1). We note that many more variables were studied and analyzed, but we are just discussing here those variables that are included in the final set of models. The decision of what variables should be in the final model, is based on econometric tests as well as on basic information provided by the Navy.

As was discussed above, this is a two-step model. . We need to prepare the data according to the basic Navy rules. These rules are defined in terms of the Years Commissioned Service (YCS) and . promotion opportunity and selection rate and refers to the percentage of all officers selected for promotion. Based on Navy and DoD documentation, we used the following policy assumptions:

- 1. O4 timing 10 years +/- 1 year, opportunity 80% +/- 10%
- 2. O5 timing 16 years +/- 1 year, opportunity 70% +/- 10%
- 3. O6 timing 22 years +/- 1 yr, opportunity 50% +/- 10%
- 4. For O3 and O2 all qualified

Based on this, the rules we used in the analysis are:

O3 to O4: Minimum of 9 years YCS

O4 to O5: Minimum of 15 years YCS

#### O<sub>5</sub> to O<sub>6</sub>: Minimum of 21 years YCS

Finally, given that there is no stated YCS for promotion to O7, we use an additional six year window from previous rank to define it as:

O6 to O7: Minimum of 27 years YCS.

In terms of time in rank, this translates to effectively six years in rank (minimum) before an officer can be promoted to O<sub>4</sub>, O<sub>5</sub>, O<sub>6</sub> and O<sub>7</sub>.

The office data is an unbalanced panel data set, therefore, we controlled for left and right censoring. Appendix A describes all of the variables used in the final model and lists the major models investigated prior to finding the final model. We estimated the promotion probabilities for each one of the five officer skill groups. For purposes of illustration only the results for the Aviation skill group are discussed. Three sets of results are reported for each for the Aviation group: the full sample, Pre September, 2001 and Post September, 2001.

Table.2. Final list of variables (with a brief description) used in CCMT Officers Project

	Categorical/Discrete Variables
GENDER	Male = 1
RACE_**	Race (empirical flag)
PAQD_**	Primary AQD (empirical flag)
SSFUNC_**	Sub-specialty – grouped by FUNCTION (empirical flags)
FLTCONC**	Fleet concentration (empirical flags)
SS_SEA / SS_OTH	Sea, shore, or other duty.
EDUC_MAP	Education level (MA plus), rest are reference category
DC***	Designation Category Code (empirical flags)
APC_MISS	If APC score is missing
JC_*	Joint Specialty Codes (empirical flags)
SCREENING_**	Screening variables (N0=none; N1=CO_SCRN; N2=DH_SCRN; N3=MAJCMD_SCRN; and N4=XO_SCRN)
	Continuous Variables
TIR, TIR2	Time in rank, Time in rank square
YCS, YCS2	Years since commission, and its square
APC_INDEX	APC Measures (additive index based on academic profile code)
NUMAQD	Total number of AQDs (additional qualification designation) to date
AVIA_MOS_OPER_FLY	Only for aviation (Months of Operational Flying)
PROFEXP EQUALOPP MILBEAR TEAMWORK LEADER TACTPERF RS_SUMAVG	Performance evaluation variables
MTGAGE_1 NASDAQ_1 R_GDP_1 UNEMPL_1	Macro Variables (lagged one month)
MTGAGE_12L NASDAQ_12L R_GDP_12L UNEMPL_12L	Macro Variables (12 month lagged moving average), where applicable.

#### The Basic Analysis – The Aviation Skill Group Example

In this section, a summary of model estimates for the Aviation skill group is provided. All results presented here are in terms of averages for the Aviation group. The model, however, also provides the possibility of individuals' forecasts.

Two sets of results are summarized here: Pre and Post September, 2001. In each case, the two stages of the analysis are presented: Stay-Loss (Step One) and the within year promotion probabilities. Each transition in the promotion stage reflects the progression from one year to the next. The forecasting tables represent the transition probability within 1 to 7 years from each grade on.

Table 3.

Aviations – Pre September 2001

Step-One: Stay - Loss

Estimated Transition Probabilities						
	О3	04	<b>O</b> 5	06	LO	
03	0.985	0.000	0.000	0.000	0.015	
04	0.000	0.977	0.003	0.000	0.020	
05	0.000	0.000	0.979	0.001	0.020	
06	0.000	0.000	0.000	0.941	0.059	
		Actu	ual Transiti	ion Counts		
	О3	04	<b>O</b> 5	06	LO	TOTAL
03	11452	2	0	0	163	11617
04	0	3962	28	0	161	4151
<b>O</b> 5	0	0	3546	10	47	3603
06	0	0	0	1268	99	1367
		Predi	cted Trans	ition Coun	ts	
	О3	04	<b>O</b> 5	06	LO	
О3	11439	0	0	0	178	11617
04	0	4056	12	0	83	4151
<b>O</b> 5	0	0	3527	4	73	3603
06	0	0	0	1287	80	1367

Table 4.
Step-Two: Promotion Probabilities (Within a Year)

	Prior Probabilities Used							
	О3	04	<b>O</b> 5	06	LO			
О3	0.832	0.096	0.000	0.000	0.072			
04	0.000	0.836	0.111	0.000	0.053			
<b>O</b> 5	0.000	0.000	0.864	0.069	0.067			
06	0.000	0.000	0.000	0.873	0.127			
		Estimat	ed Transiti	on Probabi	ilities			
	О3	04	<b>O</b> 5	06	LO			
О3	0.672	0.163	0.000	0.000	0.165			
<b>O</b> 4	0.000	0.775	0.225	0.000	0.000			
<b>O</b> 5	0.000	0.000	0.551	0.137	0.313			
06	0.000	0.000	0.000	0.854	0.146			
		Act	ual Transit	ion Counts	6			
	О3	04	<b>O</b> 5	06	LO	TOTAL		
О3	8114	2145	0	0	2015	12274		
04	0	2631	960	0	269	3860		
<b>O</b> 5	0	0	1194	420	351	1965		
06	0	0	0	750	216	966		
		Pred	icted Trans	sition Coun	its			
	О3	04	<b>O</b> 5	06	LO			
О3	8243	2003	0	0	2028	12274		
04	0	2992	868	0	0	3860		
<b>O</b> 5	0	0	1082	269	614	1965		
06	0	0	0	825	141	966		

Table 5.
Aviation – Mean Forecasting Promotion
Probabilities for T=2, 4, 7 years forward

		PRE-	9/11		
T=2	О3	04	O5	06	LO
03	0.451	0.236	0.037	0.000	0.276
04	0.000	0.601	0.298	0.031	0.070
<b>O</b> 5	0.000	0.000	0.303	0.192	0.505
<u> </u>	0.000	0.000	0.000	0.729	0.271
LO	0.000	0.000	0.000	0.000	1.000
T=4	О3	04	<b>O</b> 5	06	LO
03	0.204	0.248	0.098	0.014	0.436
04	0.000	0.361	0.270	0.098	0.271
<b>O</b> 5	0.000	0.000	0.092	0.198	0.710
06	0.000	0.000	0.000	0.532	0.468
LO	0.000	0.000	0.000	0.000	1.000
T=7	О3	04	<b>O</b> 5	06	LO
O3	0.062	0.168	0.106	0.047	0.618
04	0.000	0.168	0.153	0.141	0.538
O5	0.000	0.000	0.015	0.142	0.842
06	0.000	0.000	0.000	0.331	0.669
LO	0.000	0.000	0.000	0.000	1.000
FROM_03	TO_O3	TO_04	TO_05	TO_06	LOSS
YR1	0.672	0.163	0.000	0.000	0.165
YR2	0.451	0.236	0.037	0.000	0.276
YR3	0.303	0.257	0.073	0.005	0.362
YR4	0.204	0.248	0.098	0.014	0.436
YR5	0.137	0.226	0.110	0.026	0.502
YR6	0.092	0.197	0.111	0.037	0.563
YR7	0.062	0.168	0.106	0.047	0.618
FROM_O4	TO_03	TO_04	TO_05	TO_06	LOSS
YR1	0.000	0.775	0.225	0.000	0.000
YR2	0.000	0.601	0.298	0.031	0.070
YR3	0.000	0.466	0.299	0.067	0.168
YR4	0.000	0.361	0.270	0.098	0.271
YR5	0.000	0.280	0.230	0.121	0.370
YR6	0.000	0.217	0.189	0.135	0.459
YR7	0.000	0.168	0.153	0.141	0.538
FROM_O5	TO_03	TO_04	TO_05	TO_06	LOSS
YR1	0.000	0.000	0.551	0.137	0.313
YR2	0.000	0.000	0.303	0.192	0.505
YR3	0.000	0.000	0.167	0.206	0.628
YR4	0.000	0.000	0.092	0.198	0.710
YR5	0.000	0.000	0.051	0.182	0.767
YR6	0.000	0.000	0.028	0.162	0.810
YR7	0.000	0.000	0.015	0.142	0.842

Table 6.
Aviation – Post September 2001
Step-One: Stay Loss

Estimated Transition Probabilities							
	03	O4	05	-100abilitie 06	es LO		
O3	0.985	0.000	0.000	0.000	0.015		
04	0.000	0.950	0.003	0.000	0.047		
05	0.000	0.000	0.983	0.000	0.017		
06	0.000	0.000	0.000	0.940	0.060		
	Actual Transition Counts						
	О3	04	<b>O</b> 5	06	LO	TOTAL	
03	7137	8	0	0	75	7220	
04	0	5343	3	0	317	5663	
05	0	0	3410	12	36	3458	
06	0	0	0	1351	86	1437	
		Predicted	d Transitio	n Counts			
	03	04	<b>O</b> 5	06	LO		
03	7112	0	0	0	108	7220	
04	0	5379	19	0	265	5663	
05	0	0	3398	0	60	3458	
06	0	0	0	1351	86	1437	

Table 7.
Step-Two: Promotion Probabilities (Within a Year)

	Estimated Transition Probabilities					
	03	04	05	06	LO	
О3	0.591	0.276	0.000	0.000	0.133	
04	0.000	0.727	0.229	0.000	0.044	
<b>O</b> 5	0.000	0.000	0.710	0.140	0.150	
06	0.000	0.000	0.000	0.772	0.228	
Actual Transition Counts						
	03	04	<b>O</b> 5	06	LO	TOTAL
О3	4224	2220	0	0	973	7417
04	0	3196	1261	0	285	4742
<b>O</b> 5	0	0	1893	436	349	2678
06	0	0	0	604	235	839
Predicted Transition Counts						
	03	04	05	06	LO	
О3	4385	2044	0	0	988	7417
04	0	3450	1085	0	207	4742
05	0	0	1902	374	403	2678
06	0	0	0	647	192	839

Table 8.

Aviation – Mean Forecasting Promotion

Probabilities T=2, 4, 7 years forward

POST-9/11					
T=2	03	04	O5	06	LO
О3	0.350	0.364	0.063	0.000	0.224
04	0.000	0.529	0.329	0.032	0.110
<b>O</b> 5	0.000	0.000	0.504	0.207	0.289
06	0.000	0.000	0.000	0.596	0.404
LO	0.000	0.000	0.000	0.000	1.000
T=4	О3	04	<b>O</b> 5	06	LO
03	0.122	0.319	0.173	0.025	0.360
04	0.000	0.280	0.340	0.104	0.276
<b>O</b> 5	0.000	0.000	0.254	0.227	0.518
06	0.000	0.000	0.000	0.355	0.645
LO	0.000	0.000	0.000	0.000	1.000
T=7	О3	04	<b>O</b> 5	06	LO
О3	0.025	0.167	0.191	0.075	0.542
04	0.000	0.108	0.221	0.146	0.525
<b>O</b> 5	0.000	0.000	0.091	0.163	0.746
06	0.000	0.000	0.000	0.163	0.837
LO	0.000	0.000	0.000	0.000	1.000
FROM_03	TO_03	TO_O4	TO_O5	TO_06	LOSS
YR1	0.591	0.276	0.000	0.000	0.133
YR2	0.350	0.364	0.063	0.000	0.224
YR3	0.207	0.361	0.128	0.009	0.296
YR4	0.122	0.319	0.173	0.025	0.360
YR5	0.072	0.266	0.196	0.043	0.422
YR6	0.043	0.213	0.200	0.061	0.483
YR7	0.025	0.167	0.191	0.075	0.542
FROM_O4	TO_O3	TO_O4	TO_05	TO_06	LOSS
YR1	0.000	0.728	0.229	0.000	0.044
YR2	0.000	0.529	0.329	0.032	0.110
YR3	0.000	0.385	0.355	0.071	0.190
YR4	0.000	0.280	0.340	0.104	0.276
YR5	0.000	0.204	0.306	0.128	0.363
YR6	0.000	0.148	0.264	0.141	0.447
YR7	0.000	0.108	0.221	0.146	0.525
FROM_O5	TO_03	TO_O4	TO_05	TO_06	LOSS
YR1	0.000	0.000	0.710	0.140	0.150
YR2	0.000	0.000	0.504	0.207	0.289
YR3	0.000	0.000	0.358	0.230	0.412
YR4	0.000	0.000	0.254	0.227	0.518
YR5	0.000	0.000	0.181	0.211	0.609
YR6	0.000	0.000	0.128	0.188	0.684
YR7	0.000	0.000	0.091	0.163	0.746

The above results demonstrate the capabilities of the model used. Finally, for comparison, the estimated transitions are converted into mean career paths for each

skill group. The Aviation and SWO are presented below. It is interesting to see the differences in career paths between the Pre and Post 2001 ("peace" vs. "war" periods) and among the two skill groups presented here.

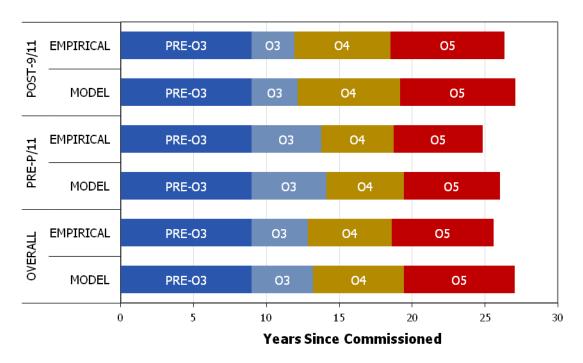


Figure 1. Career Paths, Skill Group: AVI

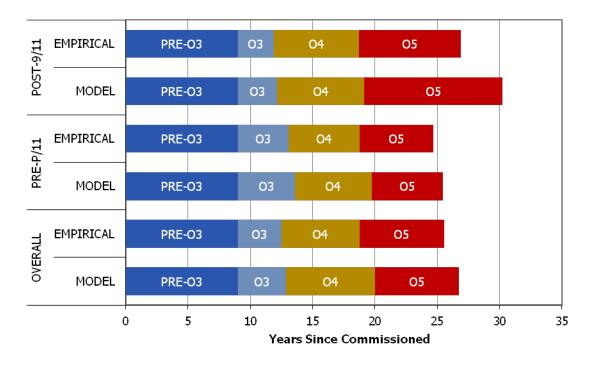


Figure 2. Career Paths, Skill Group: SWO

#### **Concluding Remarks**

The promotion probability Officers project is an experimental project where the objective was to model the promotion structure of Navy officers, by skill groups, conditional on their personal characteristics and past performance as well as on other economic and policy information. The model developed is a two stage Markov model. In the first stage the Stay-Loss probabilities are analyzed. Then, in the second stage (and conditional on the first stage) the promotion probability for each officer is estimated. The analysis is broken into two mutually exclusive periods: Before September 2001 and after that date. This allows us to capture the changes in behavior and policies as a transition for peaceful period to a war era. For comparison reasons, the analysis of the full period of the data (with a dummy variable for September, 2001) is provided.

The econometric model develop is an information theoretic, generalized maximum entropy model. This model is nonparametric in the distribution, allows us to incorporate prior information and is flexible enough to analyze small data.

In the project we were able to estimate the individual's (officer) promotion trajectory, and to investigate the changes to that trajectory subject to different constraints. These constraints can be individual-level restrictions, such as geographic requests, or global requirements, such as economic or policy changes. The model developed performs well with the data analyzed.

Additional analysis could include (i) improvement of the forecasts by allowing for additional information (variables) in the data sets (especially education and performance related information); (ii) expansion to additional officers' skill groups, and (iii) perform more simulations on different policy, economic and performance scenarios. In addition, an analysis of gender, race and other sub groups of interest should be performed.

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### Appendix A: Variables and Models

In this Appendix, the final set of models, and variables, for each skill group is summarized.

#### The Variables:

- 1. Gender (Male=1)
- 2. Race (Empirical dummies, white as reference category).
- 3. Fleet Concentration (Empirically defined dummy variables)
- 4. Sea/Shore and Other category
- 5. Education (all MA and higher education = 1; the rest are o)
- 6. APC
  - a. APC\_MISS (if APC is missing)
  - b.  $APC_INDEX = 1 (APC_Q/5 + APC_M/6 + APC_T/5)/3$  (defined only where APC MISS = 0)
- 7. Joint Specialty Code (empirically defined dummy variables)
- 8. POST911 (1 after 9/11 and 0 before)
- 9. Additional Qualification Designation (AQD)
  - a. Primary AQD (empirical dummies) identifies the most relevant AQD at current observation level.
  - b. Number of AQDs (cumulative count of the number of AQDs accumulated to date.
- 10. Subspecialty code (at the function level). Empirical dummies.
- 11. Screening (empirically defined dummy variables for category 1, 2, 3 and 4. Reference category is when Screening is missing (not screened for anything). Due to special issues with AVI data, category 2 is the reference for AVI.
- 12. Time in Rank and Time in Rank Squared
- 13. Years Since Commissioned and Years Since Commissioned Squared
- 14. Individual evaluations:
  - a. PROFEXP
  - b. EQUALOPP
  - c. MILBEAR
  - d. TEAMWORK
  - e. LEADER
  - f. TACTPERF
- 15. Rater Average Evaluations (RS\_SUMAVG)
- 16. Macro Variables (Lagged 1 period)
  - a. Mortgage Rate
  - b. NASDAQ
  - c. Real GDP
  - d. Unemployment Rate
- 17. Number of Aviation Hours accumulated (specific to AVI data set only).
- 18. Recall, this is a two-step (or two basic "periods") model. The first step model is for a period when individuals are not supposed to be promoted. During that period individuals can either stay or leave (loss) the Navy. A very small number of individuals do get promoted during that period. The next period is the phase where individuals are allowed to be promoted. They can stay at the same grade, be promoted, or leave every year. The distinction between the first and second stage is made with a dummy variable defined as follows:

```
i. @O3, INRULE = 1 if YCS>=9,
ii. @O4, INRULE = 1 if YCS>=15,
iii. @O5, INRULE = 1 if YCS>=21,
iv. @O6, INRULE = 1 if YCS>=27
```

On an average this corresponds to about 6 years in rank before promotion. This takes into account the DoD policy plus/minus one year (for flexibility).

The final model used is described in the text and provided with the CD. Here we discuss the major models and experiments done during the empirical investigations toward the final model.

#### The Basic Models Tested and Discarded:

- Models with different APC measures (continuous variables and dummy variables). In addition to the APC\_INDEX variables described above, we also estimated models with
  - b. APC\_Q, APC\_M, and APC\_T variables included as independent variables. These APC measures (Quality, Math, and Technical) were treated as continuous measures in this version.
  - c. High, Medium, and Low dummy variable versions of the APC\_Q, APC\_M, and the APC\_T variables were also tried separately. In both cases, these measures were deemed to be insignificant in both sets of models.
  - d. Although this does not constitute a model, we mined the data to see whether the promotion ratios varied significantly across the APC categories within Q, M, and T. We found that they did not. This led us to ultimately drop the dummy variable version of the model (described in the previous paragraph).
- 2. Models with the average evaluation rating (TRAITAVG) included as a predictor. Invariably, this measure was highly correlated with the constituent components so it was eventually dropped.
- 3. Models that included past promotion recommendations (for previous ranks) to capture potential unobserved propensity. This however, is inconsistent with Navy policy and behavior (i.e., according to Navy researchers, there is no reason to believe that past recommendations have any bearing on current promotion prospects). Therefore, we did not use this set of variables.
- 4. Models with finer tunings of the education dummy variables. However, since the distribution in both data sets (Stay-Loss stage and Promotion stage) was practically bi-model (clustering on BA and MA), the only two meaningful categories we could analyze are dummy variables corresponding to BA and MA.
- 5. Finally, because APC is missing for many individuals in the data (e.g., roughly 33% of cases and 36% of individuals in SWO) we tried estimating these models based only on the subset of the data where APC was observed. The qualitative findings of the models did not change.

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